

# Chiral partner structure of heavy baryons from the bound state approach with hidden local symmetry

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The chiral partner structure of heavy baryons is studied using the bound state approach by binding the heavy-light mesons to nucleon as soliton in an effective Lagrangian for the pseudoscalar and vector mesons based on the hidden local symmetry. In the heavy-light meson sector, we regard the  $H$ -doublet and  $G$ -doublet as chiral partners and couple them to light meson with minimal derivative. We find that the chiral partner of  $\Lambda_c(\frac{1}{2}^+, 2286)$  is the  $\Lambda_c(\frac{1}{2}^-, \frac{3}{2}^-)$  heavy quark doublet with the mass of about 3.1 GeV but not  $(\Lambda_c(\frac{1}{2}^-, 2595), \Lambda_c(\frac{3}{2}^-, 2625))$  which might be interpreted as an orbital excitation of the  $\Lambda_c(\frac{1}{2}^+, 2286)$ . The same model is applied to the bottom sector, and the chiral partner of  $\Lambda_b(\frac{1}{2}^+, 5625)$  is shown to have the mass of about 6.5 GeV. The chiral partner structures for the isospin vector heavy baryons are also discussed. For the pentaquark states, we find that the masses of the pentaquark state made of ground state heavy-light meson and its chiral partner are similar, and both of them are below the  $Dp$  threshold, which therefore cannot be ruled out by the present data.

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## I. INTRODUCTION

Chiral symmetry plays an important role in hadron physics. When we set  $N_f$  flavor light quarks to be massless, QCD has an  $SU(N_f)_L \times SU(N_f)_R$  chiral symmetry at Lagrangian level. The chiral symmetry is not preserved by QCD vacuum but broken dynamically to its vector component, i.e.,  $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ . This dynamical chiral symmetry breaking splits the degeneracy of the chiral partners, which are supposed to be degenerate when the chiral symmetry is restored. So that the study of the chiral partner structure of hadrons can help us to reveal the magnitude of the chiral symmetry breaking, i.e., the order parameter.

In the light meson sector, the chiral partner structure might be complicated since two of light quarks are involved (see e.g. Refs. [1, 2]). On the other hand, studying the chiral partner structure in the heavy-light meson sector would be easier since they include only one light quark, the dynamics of which is controlled by the chiral symmetry. In addition to the chiral symmetry, the dynamics

of the heavy-light mesons is also controlled by spin-flavor symmetry due to their heavy quark constituent [3]. Based on this heavy quark symmetry, the ground states form a doublet with spin-parity quantum numbers  $(1^-, 0^-)$ , and the first excited states belong to  $(1^+, 0^+)$  doublet. In Ref. [4], it was proposed that these two doublets are chiral partner to each other in the QCD-like models. As a signature of the chiral partner structure, the mass splitting of them is induced by the dynamical breaking of the chiral symmetry, so that the mass difference is about the constituent quark mass. This was confirmed by the spectrum of the relevant particles,  $m_{D_0^*} - m_D \simeq m_{D_1} - m_{D^*} \simeq 450$  MeV is at the same order of  $m_{D_{s0}}(2317) - m_{D_s} \simeq m_{D_{s1}}(2460) - m_{D_s^*} \simeq 350$  MeV (see e.g. Refs. [5, 6]).

In Ref. [7], the analysis on the chiral partner structure of heavy baryons was made based on the bound state picture [8, 9] together with the heavy quark symmetry in which the heavy baryon is introduced [7, 10–16] as the heavy mesons bound with the nucleon as the soliton. In the analysis, the excited heavy baryon  $\Lambda_c(2595)$  with  $J^P = \frac{1}{2}^-$  is regarded as the chiral partner to the ground state baryon  $\Lambda_c(2268)$  ( $J^P = \frac{1}{2}^+$ ).

In this paper, we revisit the chiral partner structure of the heavy baryons in the bound state approach based on our recent progress in the soliton property [17, 18] and the effective Lagrangian for the heavy-light meson chiral partner structure [19, 20]. In the light meson sector, we considered all the  $\mathcal{O}(N_c)$  terms of hidden local symmetry (HLS), all the  $\mathcal{O}(p^2)$ ,  $\mathcal{O}(p^4)$  and homogeneous Wess-Zumino (hWZ) terms [2, 21]. In the heavy and light meson interaction sector, we start with the interaction Lagrangian for heavy-light meson and light mesons analyzed in Ref. [19, 20], where the chiral partner is introduced in the framework of a linear sigma model. We integrate out the scalar mesons and integrate in the vector mesons to construct an effective Lagrangian for heavy mesons interacting with the pseudoscalar mesons and vector mesons based on the hidden local symmetry (HLS) [2, 21] (See, e.g. Refs. [22] for alternative approaches.), and the heavy quark symmetry. Then, we consider that the static soliton couples to the heavy-light meson and study their spectrum. After the derivation of the heavy baryon spectrum in the static case, we consider the collective coordinate quantization to make states definite quantum numbers. Our explicit calculation shows that, in the heavy quark limit and large  $N_c$  limit, up to the  $\mathcal{O}(p^4)$  terms of HLS, the chiral partner of  $\Lambda_c(\frac{1}{2}^+, 2286)$  is predicted to be the  $\Lambda_c(\frac{1}{2}^-, \frac{3}{2}^-)$  heavy quark doublet with the mass of about 3.1 GeV but not  $(\Lambda_c(\frac{1}{2}^-, 2595), \Lambda_c(\frac{3}{2}^-, 2625))$  listed in the PDG tabel [23], which might be interpreted as an orbital excitation of the  $\Lambda_c(\frac{1}{2}^+, 2286)$ . Extending our approach to bottom sector we predicted the chiral partner structure of bottom baryons. We finally studied the pentaquark spectrum using our framework and found that the masses of the pentaquark states made of ground state heavy-light meson and its chiral partner are similar, and both of them are

below the  $Dp$  threshold, which therefore cannot be ruled out by the present data [24].

This paper is organized as follows: In Sec. II, the chiral partner structure of heavy baryons is studied. We derive the analytic forms of the heavy baryons masses. The heavy baryon spectrum with chiral partner structure is estimated in Sec. III. And the pentaquark states spectrum is estimated in Sec. IV. The last section is for a summary and discussions. Some useful explicit derivations are given in Appendix.

## II. HEAVY BARYONS IN THE EFFECTIVE LAGRANGIAN FOR HEAVY-LIGHT MESONS WITH CHIRAL DOUBLING

### A. Effective Lagrangian for heavy-light mesons with chiral doubling

Here, we construct the effective Lagrangian describing the interaction between the heavy-light mesons and the light mesons. With respect to the chiral transformation property of the light quarks in the heavy-light mesons, the heavy-light meson field can be decomposed into a right-handed component  $\mathcal{H}_R$  and left-handed one  $\mathcal{H}_L$  [4]. They, under chiral  $SU(2)_L \times SU(2)_R$  symmetry, transform as

$$\mathcal{H}_L \rightarrow \mathcal{H}_L g_L^\dagger, \quad \mathcal{H}_R \rightarrow \mathcal{H}_R g_R^\dagger. \quad (1)$$

In Ref. [19, 20] these fields are used to construct a Lagrangian where the chiral symmetry are realized linearly. In the present paper for the study of the heavy baryon spectrum in the bound state approach, we adopt the non-linear realization of the chiral symmetry. Then, by replacing  $M$  in Eq. (29) of Ref. [19, 20] with  $F_\pi U$  where  $U = e^{2i\pi/F_\pi}$ , we obtain

$$\begin{aligned} \mathcal{L}_{\text{heavy}} = & \frac{1}{2} \text{Tr} [\bar{\mathcal{H}}_L i(v \cdot \partial) \mathcal{H}_L] + \frac{1}{2} \text{Tr} [\bar{\mathcal{H}}_R i(v \cdot \partial) \mathcal{H}_R] - \frac{\Delta}{2} \text{Tr} [\bar{\mathcal{H}}_L \mathcal{H}_L + \bar{\mathcal{H}}_R \mathcal{H}_R] \\ & - \frac{g_\pi F_\pi}{4} \text{Tr} [U^\dagger \bar{\mathcal{H}}_L \mathcal{H}_R + U \bar{\mathcal{H}}_R \mathcal{H}_L] \\ & + i \frac{g_A}{2} \text{Tr} [\gamma^5 \gamma^\mu \partial_\mu U^\dagger \bar{\mathcal{H}}_L \mathcal{H}_R - \gamma^5 \gamma^\mu \partial_\mu U \bar{\mathcal{H}}_R \mathcal{H}_L], \end{aligned} \quad (2)$$

where  $\Delta, g_\pi$  and  $g_A$  are parameters. In the present work, we include the vector mesons using the HLS [2, 21] by introducing the matrix valued variables  $\xi_L$  and  $\xi_R$  as  $U = \xi_L^\dagger \xi_R$ . Then, similarly to Ref. [6] we convert the heavy meson fields as

$$\hat{\mathcal{H}}_L = \mathcal{H}_L \xi_L^\dagger, \quad \hat{\mathcal{H}}_R = \mathcal{H}_R \xi_R^\dagger, \quad (3)$$

which, under the full symmetry transformation  $G_{\text{full}} = [SU(2)_L \times SU(2)_R]_{\text{chiral}} \times [U(2)]_{\text{HLS}}$ , transform as

$$\hat{\mathcal{H}}_L \rightarrow \hat{\mathcal{H}}_L h^\dagger(x), \quad \hat{\mathcal{H}}_R \rightarrow \hat{\mathcal{H}}_R h^\dagger(x). \quad (4)$$

Associated with the field redefinitions in Eq. (3), it is convenient to use the following quantities for the  $\pi$  fields:

$$\begin{aligned} \hat{\alpha}_{\parallel\mu} &= \frac{1}{2i} \left( D_\mu \xi_R \cdot \xi_R^\dagger + D_\mu \xi_L \cdot \xi_L^\dagger \right), \\ \hat{\alpha}_{\perp\mu} &= \frac{1}{2i} \left( D_\mu \xi_R \cdot \xi_R^\dagger - D_\mu \xi_L \cdot \xi_L^\dagger \right), \end{aligned} \quad (5)$$

where the covariant derivative  $D_\mu$  is given by  $D_\mu = \partial_\mu - iV_\mu$  with  $V_\mu$  being the gauge field of the HLS. By using these quantities, the above Lagrangian is extended to include the vector mesons as

$$\begin{aligned} \mathcal{L}_{\text{heavy}} &= \frac{1}{2} \text{Tr} \left[ \hat{\mathcal{H}}_L (iv \cdot \tilde{D}) \tilde{\mathcal{H}}_L \right] + \frac{1}{2} \text{Tr} \left[ \hat{\mathcal{H}}_R (iv \cdot \tilde{D}) \tilde{\mathcal{H}}_R \right] - \frac{\Delta}{2} \text{Tr} \left[ \tilde{\mathcal{H}}_L \hat{\mathcal{H}}_L + \tilde{\mathcal{H}}_R \hat{\mathcal{H}}_R \right] \\ &\quad - \frac{g_\pi F_\pi}{4} \text{Tr} \left[ \tilde{\mathcal{H}}_L \hat{\mathcal{H}}_R + \tilde{\mathcal{H}}_R \hat{\mathcal{H}}_L \right] - g_A \text{Tr} \left[ \gamma^5 \gamma^\mu \hat{\alpha}_{\perp\mu} \left( \tilde{\mathcal{H}}_L \hat{\mathcal{H}}_R + \tilde{\mathcal{H}}_R \hat{\mathcal{H}}_L \right) \right], \end{aligned} \quad (6)$$

where  $\tilde{D}$  is defined as  $\tilde{D}_\mu = \partial_\mu - iV_\mu - i\kappa\alpha_{\parallel\mu}$  with  $\kappa$  being a real parameter measuring the magnitude of the violation of the vector meson dominance.

To study the chiral partner structure of the heavy baryons, we rewrite the Lagrangian (6) in terms of the heavy-light meson doublets  $\hat{H}$  and  $\hat{G}$  with quantum numbers  $\hat{H} = (0^-, 1^-)$  and  $\hat{G} = (0^+, 1^+)$ , specifically, make the substitution

$$\hat{\mathcal{H}}_L = \frac{1}{\sqrt{2}} [\hat{G} - i\hat{H}\gamma_5], \quad \hat{\mathcal{H}}_R = \frac{1}{\sqrt{2}} [\hat{G} + i\hat{H}\gamma_5]. \quad (7)$$

In terms of the physical states, the  $\hat{H}$  and  $\hat{G}$  doublets can be explicitly expressed as

$$\hat{H} = \frac{(1 + \not{p})}{2} [D^{*\mu} \gamma_\mu + iD\gamma_5], \quad \hat{G} = \frac{(1 + \not{p})}{2} [-D_1'^{\mu} \gamma_\mu \gamma_5 + D_0^*]. \quad (8)$$

Substituting Eq. (7) into Eq. (6), we obtain

$$\begin{aligned} \mathcal{L}_{\text{heavy}} &= -\text{Tr} \left[ \hat{G} (iv \cdot \tilde{D}) \tilde{G} \right] + \text{Tr} \left[ \hat{H} (iv \cdot \tilde{D}) \tilde{H} \right] - \frac{\Delta}{2} \text{Tr} \left[ \hat{G} \tilde{G} - \hat{H} \tilde{H} \right] \\ &\quad - \frac{g_\pi F_\pi}{4} \text{Tr} \left[ \hat{H} \tilde{H} + \hat{G} \tilde{G} \right] + g_A \text{Tr} \left[ \hat{H} \gamma_\mu \gamma_5 \hat{a}_\perp^\mu \tilde{H} \right] - g_A \text{Tr} \left[ \hat{G} \gamma_\mu \gamma_5 \hat{a}_\perp^\mu \tilde{G} \right]. \end{aligned} \quad (9)$$

This expression explicitly shows that the  $g_\pi F_\pi$  term splits the spectrum of  $\hat{H}$  and  $\hat{G}$  doublets while the  $\Delta$  term shifts the masses of these two doublets toward the same direction. In the present paper, we use the physical masses of heavy mesons as inputs to calculate the heavy baryon masses, so that we drop the  $g_\pi F_\pi$  term and the  $\Delta$  term in the following calculation of the masses of heavy baryons. Note that, due to the chiral partner structure adopted here, the magnitudes of the coupling constants in the last two terms of Eq. (9) are the same therefore the chiral partner spectrum is predictable.

## B. Heavy baryon masses from the bound state approach

In this subsection we derive the heavy baryon masses based on the bound state approach [7, 12–16].

To make the mesonic theory to be a baryonic one, we follow the standard procedure to take the Hedgehog ansätze for a classical soliton [25]

$$\xi_R = \xi_L^\dagger = \xi_c(\mathbf{x}) = \exp \left[ i\boldsymbol{\tau} \cdot \hat{\mathbf{x}} \frac{F(x)}{2} \right] , \quad (10)$$

with  $\tau_i$  as the Pauli matrices and the subscript  $c$  standing for the classical solution. From the Hedgehog ansätze (10) one can easily see that  $\xi_c$  transforms under separate spatial rotation and isospin rotation but is invariant under the combined rotation, i.e., the Hedgehog profile correlates the angular momentum and the isospin. For the vector mesons, their profile functions can be parameterized as [26, 27]

$$\omega_{\mu,c} = \omega(x)\delta_{0\mu}, \quad \rho_{i,c}^a = \frac{1}{gx}\epsilon_{ija}\hat{\mathbf{x}}_j G(x), \quad \rho_{0,c}^a = 0. \quad (11)$$

From the hedgehog ansätze (10) and profile functions (11) we express the quantities  $\hat{\alpha}_\perp^\mu$  and  $\hat{\alpha}_\parallel^\mu$  as

$$\hat{\alpha}_\perp^\mu = (0, \mathbf{a}_\perp), \quad \hat{\alpha}_\parallel^\mu = (a_\parallel, \mathbf{a}_\parallel), \quad (12)$$

where

$$\begin{aligned} \mathbf{a}_\perp &= \frac{1}{2} \left[ \frac{\sin F(x)}{x} \boldsymbol{\tau} + \left( F'(x) - \frac{\sin F(x)}{x} \right) (\boldsymbol{\tau} \cdot \hat{\mathbf{x}}) \hat{\mathbf{x}} \right] \\ a_\parallel &= -\frac{g}{2}\omega(x), \\ \mathbf{a}_\parallel &= \left[ \frac{1}{x} \sin^2 \frac{F}{2} - \frac{1}{2x} G(x) \right] \hat{\mathbf{x}} \times \boldsymbol{\tau}. \end{aligned} \quad (13)$$

In the rest frame of the heavy-light meson, i.e.,  $v_\mu = (1, \mathbf{0})$ , the  $\hat{H}$  doublet has nonvanishing elements only in the upper-right  $2 \times 2$  subblock while the  $\hat{G}$  doublet has nonvanishing elements only in the upper-left  $2 \times 2$  subblock. The matrix forms of  $\hat{H}$  and  $\hat{G}$  doublets become

$$\hat{H} = \begin{pmatrix} 0 & \mathbb{H} \\ 0 & 0 \end{pmatrix}, \quad \hat{G} = \begin{pmatrix} \mathbb{G} & 0 \\ 0 & 0 \end{pmatrix}, \quad \bar{\hat{H}} = \begin{pmatrix} 0 & 0 \\ -\mathbb{H}^\dagger & 0 \end{pmatrix}, \quad \bar{\hat{G}} = \begin{pmatrix} \mathbb{G}^\dagger & 0 \\ 0 & 0 \end{pmatrix}. \quad (14)$$

Then, the Lagrangian (9) is reduced to

$$\begin{aligned} \mathcal{L}_{\text{heavy}} &= -\text{Tr} \left[ \mathbb{G} i \partial_0 \mathbb{G}^\dagger \right] - \text{Tr} \left[ \mathbb{H} i \partial_0 \mathbb{H}^\dagger \right] \\ &\quad - \frac{1}{2}(1 + \kappa) g \omega(r) \text{Tr} \left[ \mathbb{G} \mathbb{G}^\dagger \right] - \frac{1}{2}(1 + \kappa) g \omega(r) \text{Tr} \left[ \mathbb{H} \mathbb{H}^\dagger \right] \end{aligned}$$

$$-g_A \text{Tr} [\mathbb{H} \boldsymbol{\sigma} \cdot \mathbf{a}_\perp \mathbb{H}^\dagger] + g_A \text{Tr} [\mathbb{G} \boldsymbol{\sigma} \cdot \mathbf{a}_\perp \mathbb{G}^\dagger] . \quad (15)$$

Since the Hedgehog ansatz for the Skyrme soliton correlates the angular momentum and isospin, the bound states should be invariant under the “grand spin” rotation with the operator defined as

$$\mathbf{G} = \mathbf{r} + \mathbf{J} + \mathbf{I}_{\text{light}}, \quad (16)$$

where  $\mathbf{r}$ ,  $\mathbf{J}$  and  $\mathbf{I}_{\text{light}}$  are the ordinary orbital angular momentum between the soliton and heavy-light meson, heavy meson spin, and the heavy meson isospin operators. Taking into account that the heavy quark spin is conserved in the heavy quark limit, one simply defines the “light quark grand spin” operator

$$\mathbf{g} = \mathbf{r} + \mathbf{J}_{\text{light}} + \mathbf{I}_{\text{light}}, \quad (17)$$

with  $\mathbf{J}_{\text{light}}$  as the spin operator of the light degree of freedom of the heavy-light meson, and in both  $H$  and  $G$  doublets, the eigenvalue of the operator  $\mathbf{J}_{\text{light}}$  is 1/2. So that the eigenmodes of the heavy baryons can be classified by the third component of heavy quark spin  $s_Q$  and the light quark grand spin  $(g, g_3)$  and the parity  $P$ .

Taking into account the isospin, light quark spin and heavy quark spin indices that the heavy-light meson has, one can write the static wave functions of the heavy-light mesons as [11, 13]

$$\begin{aligned} \mathbb{H}_{c, lh}^{\dagger, a} &= u^{(H)}(\mathbf{x})(\boldsymbol{\tau} \cdot \hat{\mathbf{x}})_{ad} \psi_{dl}^{(H)}(g, g_3; r, k) \chi_h^{(H)}, \\ \mathbb{G}_{c, lh}^{\dagger, a} &= u^{(G)}(\mathbf{x})(\boldsymbol{\tau} \cdot \hat{\mathbf{x}})_{ad} \psi_{dl}^{(G)}(g, g_3; r, k) \chi_h^{(G)}, \end{aligned} \quad (18)$$

where  $a, l$  and  $h$  denote the indices for the isospin of the heavy-light meson, the spin of the light degree of freedom, and the heavy quark spin, respectively.  $k$  is the eigenvalue of the operator

$$\mathbf{K} = \mathbf{I}_{\text{light}} + \mathbf{J}_{\text{light}}, \quad (19)$$

with  $k_3$  as its third component.  $\chi_h^{(H,G)}$  is factorized out due to the conservation of the heavy quark spin.  $u(x)$  is a radial function which is strongly peaked at the origin and normalized as  $\mathbf{x}^2 |u(\mathbf{x})|^2 \simeq \delta^3(\mathbf{x})$  [12, 13]. This implies that the relevant matrix elements are independent of the quantum number  $r$  [14]. The generalized “angular” wave function  $\psi_{dl}^{(H,G)}(g, g_3; r, k)$  can be expanded as [14]

$$\psi_{dl}^{(H,G)}(g, g_3; r, k) = \sum_{r_3, k_3} C_{r_3, k_3; g_3}^{r, k; g} Y_r^{r_3} \xi_{dl}(k, k_3), \quad (20)$$

where  $Y_r^{r_3}$  stands for the standard spherical harmonic representing orbital angular momentum  $r$  while  $C$  denotes the ordinary Clebsch-Gordan coefficients.  $\xi_{dl}(k, k_3)$  represents a wave function in

which the “light spin” and “light isospin” referring to the “light cloud” component of the heavy meson are added vectorially to give  $\mathbf{K} = \mathbf{I}_{\text{light}} + \mathbf{J}_{\text{light}}$  with eigenvalues  $\mathbf{K}^2 = k(k+1)$ . Note that, in the present analysis  $k$  is a good quantum number since the relevant matrix elements are independent of the quantum number  $r$ . Furthermore, both the quantum numbers for the “light spin” and “light isospin” are given by  $1/2$ , so that the possible values of  $k$  are either 0 or 1. The normalization of the eigenstate  $\psi_{dl}^{(H,G)}(g, g_3; r, k)$  is

$$\int d\Omega \psi_{dl}^{(H,G)}(g, g_3; r, k) \left[ \psi_{d'l'}^{(H,G)}(g', g'_3; r', k') \right]^\dagger = \delta_{dd'} \delta_{ll'} \delta_{gg'} \delta_{g_3 g'_3} \delta_{rr'} \delta_{kk'}, \quad (21)$$

where  $\int d\Omega$  is the solid angle integration.

From Lagrangian (15), we parameterize the potential as

$$V = \begin{pmatrix} V_H & 0 \\ 0 & V_G \end{pmatrix}. \quad (22)$$

By substituting the ansatz (20),  $V_H$  and  $V_G$  are obtained as

$$\begin{aligned} V_H &= \frac{1}{2}(1 + \kappa) g\omega(r) \text{Tr} [\mathbb{H}_c \mathbb{H}_c^\dagger] + g_A \text{Tr} [\mathbb{H}_c \boldsymbol{\sigma} \cdot \mathbf{a}_\perp \mathbb{H}_c^\dagger] \\ &= \frac{1}{2}(1 + \kappa) g\omega(0) + g_A F'(0) \left[ k(k+1) - \frac{3}{2} \right], \\ V_G &= \frac{1}{2}(1 + \kappa) g\omega(r) \text{Tr} [\mathbb{G}_c \mathbb{G}_c^\dagger] - g_A \text{Tr} [\mathbb{G}_c \boldsymbol{\sigma} \cdot \mathbf{a}_\perp \mathbb{G}_c^\dagger] \\ &= \frac{1}{2}(1 + \kappa) g\omega(0) - g_A F'(0) \left[ k(k+1) - \frac{3}{2} \right]. \end{aligned} \quad (23)$$

Next, we make a quantization by a time dependent  $SU(2)$  rotation of the fields in the HLS Lagrangian in the light-quark sector as

$$\xi_c(\mathbf{x}) \rightarrow \xi(\mathbf{x}, t) = C(t) \xi_c(\mathbf{x}) C^\dagger(t), \quad V_{\mu,c}(\mathbf{x}) \rightarrow V_\mu(\mathbf{x}, t) = C(t) V_{\mu,c}(\mathbf{x}) C^\dagger(t), \quad (24)$$

where  $C(t)$  is a time dependent unitary matrix satisfying  $C(t)C(t)^\dagger = C(t)^\dagger C(t) = 1$ . Accordingly, the heavy-light meson fields are rotated as

$$\mathbb{H}(\mathbf{x}, t) = \mathbb{H}_c(\mathbf{x}) C^\dagger(t), \quad \mathbb{G}(\mathbf{x}, t) = \mathbb{G}_c(\mathbf{x}) C^\dagger(t). \quad (25)$$

This collective rotation gives an additional contribution to the Lagrangian

$$\delta \mathcal{L}_{\text{coll}} = \frac{1}{2} \mathcal{I} \Omega^2 + \mathbf{I}_{\text{light}} \cdot \boldsymbol{\Omega}, \quad (26)$$

where the angular velocity  $\Omega_i$  corresponding to the collective coordinate rotation is defined as

$$\frac{1}{2} i \boldsymbol{\tau} \cdot \boldsymbol{\Omega} \equiv C^\dagger \partial_0 C. \quad (27)$$

$\mathcal{I}$  is the moment of the inertia of the soliton configuration. By using  $\mathcal{I}$ , the light baryon masses are expressed as

$$m_b = M_{\text{sol}} + \frac{j_b(j_b + 1)}{2\mathcal{I}} , \quad (28)$$

where  $M_{\text{sol}}$  is the soliton mass and  $j_b$  is the spin of light baryon. Using the nucleon mass  $m_N$  and the Delta mass  $m_\Delta$  as inputs,  $M_{\text{sol}}$  and  $\mathcal{I}$  are given as

$$M_{\text{sol}} = \frac{5m_N - m_\Delta}{4} , \quad \mathcal{I} = \frac{3}{2(m_\Delta - m_N)} . \quad (29)$$

From Eq. (26) one obtains [8]

$$\delta\mathcal{L}_{\text{coll}} = \frac{1}{2}\mathcal{I}\Omega^2 - \chi(k)\mathbf{K} \cdot \boldsymbol{\Omega}, \quad (30)$$

where the coefficient  $\chi(k)$  is calculated as

$$\chi(k) = \begin{cases} 0 , & (k = 0) , \\ \frac{[k(k+1)+3/4-j_l(j_l+1)]}{2k(k+1)} , & (k \neq 0) \end{cases} , \quad (31)$$

with  $j_l$  being the spin quantum number of the light degree of freedom of heavy-light meson. For convenience we show the derivation in Appendix. A. The Hamiltonian of the collective rotated system is obtained by the standard Legendre transformation as

$$H_{\text{coll}} = \frac{1}{2\mathcal{I}}[\mathbf{J}^{\text{sol}} + \chi(k)\mathbf{K}]^2, \quad (32)$$

where  $\mathbf{J}^{\text{sol}}$  is the canonical momentum conjugating to the collective variable  $C(t)$ :

$$J_a^{\text{sol}} = \frac{\partial\delta\mathcal{L}^{\text{coll}}}{\partial\Omega^a} = \mathcal{I}\Omega_a + I_{\text{light}}^a. \quad (33)$$

The first term  $\mathcal{I}\Omega_a$  is the isospin operator for the light baryon sector while the second term  $I_{\text{light}}^a$  is the isospin operator for the heavy-light mesons interacting with the light baryon, so that  $\mathbf{J}^{\text{sol}}$  is identical to the isospin operator for the heavy baryon,  $\mathbf{I}$ :

$$\mathbf{J}^{\text{sol}} = \mathbf{I} . \quad (34)$$

After the collective coordinate rotation, the total spin of the light degrees of freedom in the heavy baryon is defined as

$$\mathbf{j} = \mathbf{J}^{\text{sol}} + \mathbf{g} = \mathbf{J}^{\text{sol}} + \mathbf{r} + \mathbf{K}. \quad (35)$$



By including the heavy quark spin, the spin operator for the heavy baryon is expressed as  $\mathbf{J}_B = \mathbf{j} \pm \mathbf{S}_Q$  with eigenvalues  $j_{B,\pm} = j \pm 1/2$  (in the case of  $j = 0$ , only  $j_B = 1/2$  exists). Then, we can express the collectively rotated Hamiltonian as

$$H_{\text{coll}} = \frac{1}{2\mathcal{I}} \left[ [1 - \chi(k)] \mathbf{I}^2 + \chi(k) [\chi(k) - 1] \mathbf{K}^2 + \chi(k) (\mathbf{j} - \mathbf{r})^2 \right]. \quad (36)$$

Gathering all the above contributions, we finally obtain the heavy baryon mass as

$$M_{(\text{heavy baryon})} = M_{\text{sol}} + \bar{M}_{H,G} + V_{H,G} + H_{\text{coll}}, \quad (37)$$

where  $V_H$  ( $V_G$ ) is the binding energy corresponding to the heavy meson  $H$  ( $G$ ).  $\bar{M}_{H,G}$  are the weight-averaged heavy meson masses with  $\bar{M}_H = (3m_{D^*} + m_D)/4$  and  $\bar{M}_G = (3m_{D_1} + m_{D_0^*})/4$ . Note that each combination of  $(I, j)$  generates a pair of degenerate states with  $j_{B,\pm} = j \pm 1/2$ .

### III. CHIRAL DOUBLING OF HEAVY BARYONS

In this section, we study the chiral doubling structure using the formulas obtained in the previous section. In the following analysis, we restrict ourselves to the case with  $\mathbf{r} = 0$ , and use the following values of heavy meson masses as inputs:

$$(m_D, m_{D^*}, m_{D_0^*}, m_{D_1}) = (1.86, 2.01, 2.32, 2.42) \text{ [GeV]}, \quad (38)$$

which lead to

$$(\bar{M}_H, \bar{M}_G) = (1.97, 2.40) \text{ [GeV]}. \quad (39)$$

Furthermore, to simulate the profile functions  $F(r)$  and  $\omega(r)$  which are necessary to evaluate the binding energy  $V_H$  and  $V_G$  expressed in Eq. (23) we use the following inputs:

$$(m_N, m_\Delta) = (0.94, 1.23) \text{ [GeV]}, \quad (40)$$

which yields soliton mass  $M_{\text{sol}} = 0.868 \text{ GeV}$  and the inverse of the moment of inertia  $1/\mathcal{I} = 0.193 \text{ GeV}$ . Then, using the relevant expressions from HLS up to  $\mathcal{O}(p^4)$  including the hWZ terms given in Refs. [17, 18], by taking  $F_\pi$  and  $m_\rho$  as free parameters to fit the inputs (40) we obtain  $F_\pi = 62.24 \text{ MeV}$  and  $m_\rho = 417.5 \text{ MeV}$  and the values of the profile functions at origin as

$$F'(0) = 626.1 \text{ MeV}; \quad \omega(0) = -74.5 \text{ MeV}. \quad (41)$$

Moreover, we take the parameter  $a = 2$  [2, 21] and fix the universal coupling constant  $g$  in HLS through

$$g = m_\rho / (F_\pi \sqrt{a}) = 4.74. \quad (42)$$

Let us first consider the binding energy in order to determine which channel can form bound state and calculate the spectra of the chiral partners. Using Eq. (23) we obtain the binding energy between the heavy-light mesons in  $H$  doublet and soliton as

$$V_H = -0.177(1 + \kappa) + 0.626 g_A \left[ k(k+1) - \frac{3}{2} \right] \quad [\text{GeV}] . \quad (43)$$

The value of  $g_A$  is determined through the  $D^* \rightarrow D\pi$  decay as  $|g_A| = 0.56$  [19, 20]. It does not seem possible to determine the  $\omega$  coupling constant  $\kappa$  from the available experimental data for heavy meson decay. In the case of the vector meson dominance we have  $\kappa = 0$ , therefore it is reasonable to regard  $|\kappa| \lesssim 1$ . Then we conclude that the  $k = 0$  channel gives a bound state when  $g_A > 0$ . This bound state can be naturally identified with  $\Lambda_c(\frac{1}{2}^+, 2286)$  from the quantum number, so that we assume  $g_A > 0$  in the following analysis. Since the collective energy is zero for the  $k = 0, I = 0$  state, we use the experimental value of the mass of  $\Lambda_c(\frac{1}{2}^+, 2286)$  as an input to determine the value of  $\kappa$ . Using  $M_{\Lambda_c} = 2.29 \text{ GeV}$ , we obtain  $\kappa = -0.83$ .

Next, we consider the bound states made from  $G$  doublet. The binding energy is expressed as

$$V_G = -0.177(1 + \kappa) - 0.626 g_A \left[ k(k+1) - \frac{3}{2} \right] \quad [\text{GeV}] . \quad (44)$$

As we can see easily,  $V_G > 0$  for  $k = 0$  ( $g_A > 0$ ), so that there is no bound state in the  $k = 0$  channel. For the  $k = 1$  channel, using  $\kappa = -0.83$  determined above, we obtain  $V_G = -0.205 \text{ GeV}$ , which implies that the  $k = 1$  channel is actually bound.

From Eq. (35) the total spin of the light degrees of freedom becomes 1 ( $I = 0$ ), so that the resultant heavy baryons form a heavy-quark doublet consisting  $\Lambda_c(\frac{1}{2}^-)$  and  $\Lambda_c(\frac{3}{2}^-)$ . Combined with the collective energy, the mass of the bound state is expressed as

$$M_{\Lambda_c(\frac{1}{2}^-, \frac{3}{2}^-)} = M_{\text{sol}} + \bar{M}_G + V_G + \frac{1}{4\mathcal{I}} , \quad (45)$$

which leads to

$$M_{\Lambda_c(\frac{1}{2}^-, \frac{3}{2}^-)} = 3.13 [\text{GeV}] . \quad (46)$$

This value is much larger than the experimental values for the masses of negative parity baryons;  $M(\Lambda_c(\frac{1}{2}^-, 2595)) = 2.59 \text{ GeV}$  and  $M(\Lambda_c(\frac{3}{2}^-, 2625)) = 2.63 \text{ GeV}$ . So, we conclude that the negative parity baryons found by experiments  $\Lambda_c(\frac{1}{2}^-, 2595)$  and  $\Lambda_c(\frac{3}{2}^-, 2625)$  are not the chiral partner to the ground state baryon  $\Lambda_c(\frac{1}{2}^+, 2285)$ . In the present bound state approach, they should be regarded as  $r = 1$  state made from the  $H$  doublet and nucleon. Then, we expect to have a doublet for the chiral partner around 3.1 GeV region.

TABLE I: Predicted mass for the charmed baryon. The left table is for  $H$  doublet and the right table is for  $G$  doublet.

$I$	$j$	states	$M^H(\text{MeV})$	$I$	$j$	$I(j_B^P)$	$M^G(\text{MeV})$
0	0	$\Lambda_c(\frac{1}{2}^+)$	2286.46(input)	0	1	$\Lambda_c(\frac{1}{2}^-), \Lambda_c(\frac{3}{2}^-)$	3131.66
1	1	$\Sigma_c(\frac{1}{2}^+), \Sigma_c(\frac{3}{2}^+)$	2481.13	1	0	$\Sigma_c(\frac{1}{2}^-)$	3131.66
				1	1	$\Sigma_c(\frac{1}{2}^-), \Sigma_c(\frac{3}{2}^-)$	3228.99
				1	2	$\Sigma_c(\frac{3}{2}^-), \Sigma_c(\frac{5}{2}^-)$	3423.66

TABLE II: Predicted mass for the bottom baryon. The left table is for  $H$  doublet and the right table is for  $G$  doublet.

$I$	$j$	$I(j_B^P)$	$M^H(\text{MeV})$	$I$	$j$	$I(j_B^P)$	$M^G(\text{MeV})$
0	0	$\Lambda_b(\frac{1}{2}^+)$	5625.07	0	1	$\Lambda_b(\frac{1}{2}^-), \Lambda_b(\frac{3}{2}^-)$	6470.27
1	1	$\Sigma_b(\frac{1}{2}^+), \Sigma_b(\frac{3}{2}^+)$	5819.74	1	0	$\Sigma_b(\frac{1}{2}^-)$	6470.27
				1	1	$\Sigma_b(\frac{1}{2}^-), \Sigma_b(\frac{3}{2}^-)$	6567.6
				1	2	$\Sigma_b(\frac{3}{2}^-), \Sigma_b(\frac{5}{2}^-)$	6762.27

We next study the  $I = 1$  baryons. In the positive parity baryon sector,  $k = 0$  channel is bound, so that the total spin of the light degrees of freedom in the heavy baryon becomes one. As a result, the spin of  $\Sigma_c$  baryons with positive parity is either  $1/2$  or  $3/2$ . In the mass formula in Eq. (37), only  $H_{\text{coll}}$  changes its value depending on the isospin of baryons. Since  $\chi(k) = 0$  for  $k = 0$ , the mass difference between the  $\Sigma_c$  and the  $\Lambda_c$  in the positive negative parity is obtained as

$$M_{\Sigma_c(\frac{1}{2}^+, \frac{3}{2}^+)} - M_{\Lambda_c(\frac{1}{2}^+)} = \frac{1}{\mathcal{I}}. \quad (47)$$

In the negative parity baryon sector,  $k = 1$  channel is bound, then the eigenvalue for  $\mathbf{j}$  is either 0, 1 or 2. We summarize our predicted results for the charm baryon spectrum in Table. I.

We next study the mass spectrum of bottom baryon by substituting the bottom meson masses into the charm meson masses in Eq. (37). In the bottom meson spectrum, the masses of the ground states  $B$  and  $B^*$  are well measured but masses of the mesons in the  $G$  doublets are not well established. Here, we naively estimate them using  $m_{B_0^*} - m_B = m_{D_0^*} - m_D = 2403 - 1869.6 = 533.4$  MeV and  $m_{B_1^*} - m_{B^*} = m_{D_1^*} - m_{D^*} = 2427 - 2010.25 = 416.75$  MeV which lead to  $m_{B_0^*} = 5812.9$  MeV and  $m_{B_1^*} = 5741.85$  MeV. Our numerical results for the masses of the heavy baryons including bottom quark with the corresponding quantum numbers are given in Table II.

TABLE III: Predicted mass for the pentaquark state. The left table is for  $H$  doublet and the right table is for  $G$  doublet.

$I$	$j$	Candidates	$M^{5,H_c}(\text{MeV})$	$I$	$j$	Candidates	$M^{5,G_c}(\text{MeV})$
0	1	$\Theta_c(\frac{1}{2}^+)$	2745.15	0	0	$\Theta_c(\frac{1}{2}^-)$	2791.78

#### IV. PENTAQUARKS WITH HEAVY QUARK

We next consider the pentaquark channel. Although the existence of these kind of states still needs the experimental confirmation, theoretical study of them is meaningful. For a pentaquark state, the large component of the anti-heavy quark can be projected out with the projection operator  $(1 - \not{p})/2$ . So that, in case the heavy meson is at rest, the  $H$  doublet has nonvanishing elements only in the lower-left  $2 \times 2$  subblock while the  $G$  doublet has nonvanishing elements only in the lower-right  $2 \times 2$  subblock, i.e.,

$$H = \begin{pmatrix} 0 & 0 \\ \mathbf{H} & 0 \end{pmatrix}, G = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{G} \end{pmatrix}, \bar{H} = \gamma_0 H^\dagger \gamma_0 = \begin{pmatrix} 0 & -\mathbf{H}^\dagger \\ 0 & 0 \end{pmatrix}, \bar{G} = \gamma_0 G^\dagger \gamma_0 = \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{G}^\dagger \end{pmatrix}. \quad (48)$$

Then, substituting  $v_\mu$  with  $-v_\mu$ , following the above derivation, one can see that both the binding energies given by Eqs. (23) change a sign. As a result, the binding energies for the pentaquark states made from anti- $H$  doublet and the pentaquark states made from anti- $G$  doublet are expressed as

$$\begin{aligned} V_H^5 &= 0.177(1 + \kappa) - 0.626 g_A \left[ k(k+1) - \frac{3}{2} \right] \quad [\text{GeV}], \\ V_G^5 &= 0.177(1 + \kappa) + 0.626 g_A \left[ k(k+1) - \frac{3}{2} \right] \quad [\text{GeV}]. \end{aligned} \quad (49)$$

Therefore, for anti- $H$  doublet, the  $k = 1$  channel gives the bound states with binding energy  $V_H^5 = -145.6$  MeV, while for anti- $G$  doublet, the  $k = 0$  channel gives the bound states with binding energy  $V_G^5 = -496.2$  MeV.

Substituting relevant numerical results, we obtain the spectrum of the pentaquark states. We list our results in Table. III for pentaquark states consisting anti-charm quark and Table. IV for pentaquark states consisting anti-bottom quark.

The results in Table. III show that the lightest charmed pentaquark states made of soliton and heavy-light mesons in the anti- $H$  doublet have masses about 2.75 GeV and their chiral partner made of the anti- $G$  doublet have the masses of about 2.80 GeV. Both of them are below the  $Dp$  threshold. The reason that the pentaquark states from anti- $H$  doublet and anti- $G$  doublet have the

TABLE IV: Predicted mass for the pentaquark state. The left table is for  $H$  doublet and the right table is for  $G$  doublet.

$I$	$j$	Candidates	$M^{5,H_b}(\text{MeV})$	$I$	$j$	Candidates	$M^{5,G_b}(\text{MeV})$
0	1	$\Theta_b(\frac{1}{2}^+)$	6083.76	0	0	$\Theta_b(\frac{1}{2}^-)$	6130.39

similar masses is that the binding energy of the anti- $H$  doublet is about 350 MeV smaller than that of the anti- $G$  doublet and collective rotation energy which is about 50 MeV does not contribute to the latter. With respect the status of the pentaquark search performed, these states cannot be ruled out and, since they cannot decay via strong process, their total widths should be narrow.

## V. SUMMARY AND DISCUSSIONS

We studied the chiral partner structure of heavy baryons in the bound state approach including the vector meson exchanging effects through the hidden local symmetry. We showed that, in the large  $N_c$  limit and the heavy quark limit, the ground state heavy baryon made of the ground state heavy-light meson and the nucleon has the chiral partner made of the excited heavy-light meson and the nucleon. Our explicit calculation shows that the chiral partner of  $\Lambda_c(\frac{1}{2}^+)$  is a heavy quark doublet of  $\Lambda(\frac{1}{2}^-)$  and  $\Lambda(\frac{3}{2}^-)$ . This contrasts to the perdition made in the pioneering work in Ref. [12], where the chiral partner is the singlet under the heavy quark spin transformation. Our prediction of the mass is about 3.1 GeV, which indicates that the  $\Lambda_c(\frac{1}{2}^-, 2595)$  and  $\Lambda_c(\frac{3}{2}^-, 2625)$  listed in the PDG table [23] should be interpreted as the  $r = 1$  excitation of  $\Lambda_c(\frac{1}{2}^+)$ .

We also studied the bound states in the pentaquark channel. We found that the  $k = 1$  channel forms bound states for anti- $H$  doublet ( $\Lambda_c(\frac{1}{2}^-)$ ,  $\Lambda_c(\frac{3}{2}^-)$ ), while the  $k = 0$  channel forms bound states for anti- $G$  doublet ( $\Lambda_c(\frac{1}{2}^+)$ ). It is found that the predicted masses of the pentaquark states made of both anti- $H$  doublet and anti- $G$  doublet are below the  $Dp$  threshold which cannot be ruled out by the present data [24].

In the present analysis, we take the infinite heavy soliton and heavy quark limits, so that both the soliton and heavy-light meson are sitting at the origin. This picture cannot be applied to the bound states with non-zero  $r$ . Since in the present analysis, the chiral partner of heavy  $\Lambda_c(\frac{1}{2}^+, 2286)$  has the mass of about 3.1 GeV which is a bound state of soliton and heavy-light mesons in  $G$  doublet, one can expect that it has a broad width due to the broad width of constituent  $P$ -wave mesons in  $G$  doublet. From the numerical results in Table II we conclude that the spectrum of the heavy baryons with bottom quark is consistent with PDG [23] for  $\Lambda_b$  and  $\Sigma_b$ .

It should be noted that, in the present analysis, we consider that the chiral partner to the nucleon is itself: The left-handed nucleon is the chiral partner of the right-handed nucleon, and vice versa. So that the chiral partner to the heavy baryon as the bound state of the  $H$  doublet and the nucleon is the one made of  $G$  doublet and the nucleon. This implies that the chiral partner structure of the heavy baryons in our approach arises from the chiral partner structure of the constituent heavy-light mesons. On the other hand, in the mirror scenario for the light baryon [28], the chiral partner to the nucleon is considered as  $N(1535)$ . In such a case, the full picture of the chiral partner structure of heavy baryons becomes complicated. We will not consider this scenario in the present work.

### Appendix A: Matrix element of heavy-light meson isospin operator

Using the Wigner-Eckart theorem, we can express the matrix element of heavy-light meson isospin operator  $I_{\text{light}}^a$  in terms of the matrix element of the operator  $\mathbf{K}$ , i.e.,

$$\begin{aligned}
\int d\Omega \langle \psi_{gg3}^{(i)} | \mathbf{I}_{\text{light}} | \psi_{gg3}^{(j)} \rangle &= \int d\Omega \langle \psi_{gg3}^{(i)} | \mathbf{K} | \psi_{gg3}^{(j)} \rangle \frac{\langle \psi_{gg3}^{(i)} | \mathbf{K} \cdot \mathbf{I}_{\text{light}} | \psi_{gg3}^{(j)} \rangle}{k(k+1)} \\
&= \int d\Omega \frac{\langle \psi_{gg3}^{(i)} | [\mathbf{K}^2 + \mathbf{I}^2 - \mathbf{J}_{\text{light}}^2] | \psi_{gg3}^{(j)} \rangle}{2k(k+1)} \langle \psi_{gg3}^{(i)} | \mathbf{K} | \psi_{gg3}^{(j)} \rangle \\
&= \frac{[k(k+1) + 3/4 - j_l(j_l+1)]}{2k(k+1)} \int d\Omega \langle \psi_{gg3}^{(i)} | \mathbf{K} | \psi_{gg3}^{(j)} \rangle \delta_{ij} \\
&= \chi(k) \mathbf{K},
\end{aligned} \tag{A1}$$

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